The Quantitative New Trade Model: Equilibrium and Welfare Analysis

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Motivation

- Quantitative trade models:
  - Multiple sectors and intermediates
  - Roundabout production matching IO data
  - Caliendo-Parro or Baqae-Farhi have CRS

- Extend to allow for EES:
  - New trade theory: EES due to love of variety
  - Empirical evidence for EES (recent): Costinot et. al. '19; BCDR '21; Lashkaripour and Lugovskyy '23; Bartelme et. al. '23; Breinlich et. al. '23
Examples in the Literature

- **BCDR ’21**: industrial policy (PC)
- **Bartelme et. al. ’23**: trade shocks on growth (PC)
- **Lashkaripour and Lugovskyy ’23**: industrial policy (MC)
- **Breinlich et. al. ’23**: import shocks on exports (MC)

**Special cases:**
- **Krugman and Venables ’95**: core-periphery
- **Antras et. al. ’22**: trade policy
- **Caliendo et. al. ’21**: optimal trade policy (Melitz)
- **Baqae and Farhi ’21**: local comparative statics (no trade)

**Background:**
- **KLR**: multi-sector gravity + EES, no intermediates
This Paper

- **Model:**
  - Caliendo-Parro + EES in VA or GO, Small Open Economy
  - $\varepsilon_k$ is the trade elasticity, $\theta_k$ is the scale elasticity

- **Uniqueness:**
  - Sufficient Uniqueness Condition (UC):
    \[
    \sum_s \theta_s \ell^F_{sk} \varepsilon_k < 1, \forall k
    \]
  - Without IO: $\ell^F_{kk} = 1$ and $\ell^F_{sk} = 0$ for $s \neq k$ ⇒ KLR’s condition:
    \[
    \theta_k \varepsilon_k < 1, \forall k
    \]
  - Proof is not yet complete for EES in GO and more than one sector with EES

- **Gains from Trade:**
  - With EES in VA, UC implies gains from trade
  - With EES in GO, could have losses from trade even under UC
1. Motivation

2. Model
2.1 Basic Assumptions
2.2 Equilibrium

3. Characterization of Equilibrium

4. Gains from Trade
Basic Assumptions

- Home is SOE
- \( K \) sectors indexed by \( k = 1, \ldots, K \)
- Armington assumption
- Perfect competition and sector-level EES
Basic Assumptions

\[ Q_k = \left( \alpha_k^{-\alpha_k} \prod_{s=1}^{K} \alpha_{sk}^{-\alpha_{sk}} \right) \overline{T}_k L_k^{\alpha_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk}} \]

\[ \alpha_{sk} \in [0, 1], \quad \alpha_k + \sum_s \alpha_{sk} = 1, \quad \alpha_k > 0 \]

\[ \overline{T}_k = T_k L_k^{\alpha_k \gamma_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk} \nu_k} \]

\[ \nu_k < \frac{\alpha_k}{1 - \alpha_k} \]
Basic Assumptions

▶ If $\nu_k = 0$, then

$$Q_k = \left( \alpha_k^{-\alpha_k} \prod_{s=1}^{K} \alpha_{sk}^{-\alpha_{sk}} \right) T_k \cdot \left( L_k \cdot L_k^{\gamma_k} \right)^{\alpha_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk}}$$

▶ This is EES in VA, a natural framework for technological EES

▶ If $\gamma_k = \nu_k$, then

$$\bar{T}_k = \tilde{T}_k \cdot Q_k^{\gamma_k}$$

▶ This is EES in gross output, and results from Krugman with

$$\gamma_k = \nu_k = \frac{1}{\sigma_k - 1}$$

where $\sigma_s$ is the EoS across domestic varieties
Basic Assumptions

- Trade shares differ between consumption and intermediates,

\[ \lambda^C_k(p_k) = \frac{p_k^{-\varepsilon_k}}{p_k^{-\varepsilon_k} + [p_k^C]^*-\varepsilon_k}, \quad \lambda_k(p_k) = \frac{p_k^{-\varepsilon_k}}{p_k^{-\varepsilon_k} + [p_k^I]^*-\varepsilon_k} \]

- Cobb-Douglas preferences across sectors,

\[ C_k = \lambda^C_k(p_k)e_k w\bar{L}, \quad \sum_k e_k = 1 \]

- Isoelastic export revenues in sector \( k \),

\[ X_k = E_k p_k^{-\varepsilon_k} \]
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Equilibrium: Prices

- Prices of domestic varieties given $\lambda_1, ..., \lambda_K$ and $L_1, ..., L_K$:

$$p_k \lambda_k^{1/\varepsilon_k} = P_k = w \cdot \tilde{\xi}_k \cdot \prod_s \lambda_{sk}^{\ell_{sk}^F/\varepsilon_s} \cdot \prod_s \left( T_s L_s \theta_s \right)^{-\ell_{sk}^F},$$

where

$$\theta_s \equiv \alpha_s \gamma_s + (1 - \alpha_s) \upsilon_s$$

and

$$\mathcal{L}^F \equiv (I - A(I + D_\upsilon))^{-1} \quad \text{with} \quad A \equiv \{\alpha_{sk}\}, \ D_\upsilon \equiv D \{\upsilon\}$$

capture forward linkages,

$$\ell_{sk}^F = -\partial \ln p_k / \partial \ln T_s$$
Equilibrium: Market Clearing

Market clearing condition in sector $k$ is

$$ p_k Q_k = C_k + X_k + \lambda_k \sum_s P_k Q_{ks} $$

or, using $d_k \equiv (C_k + X_k)/w$,

$$ L_k/\alpha_k = d_k + \lambda_k \sum_s \alpha_{ks} L_s / \alpha_s $$

Solving for $R_k \equiv L_k/\alpha_k$,

$$ R_k = \sum_s \tilde{\ell}^B_{ks} d_s $$

where

$$ \tilde{\ell}^B \equiv (I - D_\lambda A)^{-1} \quad \text{with} \quad D_\lambda \equiv D \{\lambda\} $$

captures backward linkages,

$$ \tilde{\ell}^B_{ks} = \partial R_k / \partial d_s $$
Equilibrium

An equilibrium is a wage $w$, prices $p$ and labor allocations $L$ that satisfy

$$p_k = \xi_k \cdot w \cdot \prod_s [\lambda_s(p_s)]^{\ell^F_{sk} - \delta_{sk}} \cdot \prod_s L_s^{-\theta_s \ell^F_{sk}}$$

$$L_k/\alpha_k = d_k(w, p_k) + \lambda_k(p_k) \sum_s \alpha_{ks} L_s/\alpha_s$$

$$\sum_k L_k = \bar{L}$$

Here $\delta_{sk}$ indicator function for $s = k$

We next (almost) show that there is a unique solution if

$$\sum_s \theta_s \ell^F_{sk} \varepsilon_k < 1, \forall k \quad \text{UC}$$
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Characterization of Equilibrium

- **Step 1:** Take $w$ and $L$ as given and focus on $p$:
  
  Show that $\text{UC} \implies$ There exists a unique $p$

  - This leads to function $p(w, L)$

- **Step 2:** Take $w$ as given, focus on $L$:
  
  Show that $\text{UC} \implies$ There exists a unique $L$

  - This leads to labor demand $L(w)$

- **Step 3:** Focus on $w$:
  
  Show that $\text{UC} \implies$ There exists a unique $w$
Steps 1 and 3 are relatively straightforward, step 2 is challenging

- The goods market clearing condition gives a mapping $L \rightarrow L'$,

$$L'_k/\alpha_k = d_k(p_k(L)) + \lambda_k(p_k(L)) \sum_s \alpha_{ks} L'_s/\alpha_s$$

- Existence is proved by showing that (given UC) this mapping stays inside a rectangular region of $\mathbb{R}^K_+$

- To show uniqueness we use the “Index Theorem”
Index Theorem

- Index at a fixed point is $+1$ ($-1$) if $1 - F'(L) > 0$ ($< 0$)
- Generalization: index is $\text{sgn} \left( \det (I - J) \right)$
- Index Theorem: sum of indices $= +1$
Index Theorem

- **Key implication:**
  \[ \det(I - J) > 0 \text{ at any fixed point} \implies \text{fixed point is unique} \]

- **Basic idea:** if a self-absorbing mapping is a *local* contraction mapping at each fixed point, then it has only one fixed point

- **Economics:** supply curve cuts demand curve from below at every goods market equilibrium
Jacobian

- **UC** $\implies \det(I - J) > 0$ or $\rho(J) < 1$ for $J = \text{Jacobian of } L \rightarrow L'$ (in logs) mapping at a fixed point.

- With no trade in intermediates,

$$J|_{D\lambda=1} = D^{-1}_L \cdot D_\alpha L^B \cdot \left\{ \frac{\partial d_k(p_k)}{\partial \ln p_r^{-\varepsilon_k}} \right\} \cdot \left\{ \frac{\partial \ln p_k^{-\varepsilon_k}}{\partial \ln L_s} \right\}$$

$$\leq D \left\{ L^B D_d \right\}^{-1} \cdot L^B \cdot D_d \cdot D_\varepsilon \left[ L^F \right]^T D_\theta \equiv \tilde{J}|_{D\lambda=1}$$

stochastic matrix

- Thus $\rho\left(\tilde{J}|_{D\lambda=1}\right) < 1$ if max row sums of $D_\varepsilon \left[ L^F \right]^T D_\theta$ are < 1, which is our UC,

$$\sum_s \theta_s \ell^E_{sk} \varepsilon_k < 1, \forall k$$
Uniqueness Condition

- In the case with EES in VA we show that the UC,

\[ \sum_s \theta_s \epsilon^F_{sk} \epsilon_k < 1, \forall k, \]

is sufficient for \( \rho(J) < 1 \) for any \( \lambda \)

- Still working on proof with EES in GO — so far we have it only for EES in one sector
Discussion

- With EES in VA ($\nu_k = 0, \forall k$) and $\gamma_k = \gamma, \forall k$:
  - $\nu_k = 0, \forall k \implies \theta_k = \gamma \alpha_k$ and $\ell^F_{sk} = \ell^B_{sk}, \forall s, k$ so UC becomes
    $$\gamma \sum_s \alpha_s \ell^B_{sk} \varepsilon_k = \gamma \varepsilon_k < 1, \forall k,$$
    where we have used $\sum_s \alpha_s \ell^B_{sk} = 1$
  - This is same UC in KLR for case without IO if $\gamma_k = \gamma, \forall k$

- With EES in GO ($\nu_k = \nu_k, \forall k$) and $\gamma_k = \gamma, \forall k$ then UC becomes
  $$\varepsilon_k \leq \frac{1}{\gamma \sum_s \ell^F_{sk}}$$

- EES in GO leads to increased amplification relative to EES in VA
\[ \varepsilon_k \leq \frac{1}{\gamma \max_k \sum_s \ell_{sk}^F} \] for US (Motor Vehicles)
<table>
<thead>
<tr>
<th>Sector</th>
<th>Max $\varepsilon_k$ (1)</th>
<th>Max $\varepsilon_k$, 10th pctile (2)</th>
<th>Max $\varepsilon_k$, US (3)</th>
<th>Max $\varepsilon_k$, Avg. IO (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2.8</td>
<td>3.3</td>
<td>4.2</td>
<td>5.3</td>
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<tr>
<td>Mining</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
<td>6.1</td>
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<tr>
<td>Textiles</td>
<td>2.2</td>
<td>2.7</td>
<td>4.4</td>
<td>3.5</td>
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<tr>
<td>Chemical Products</td>
<td>2.1</td>
<td>2.6</td>
<td>3.9</td>
<td>3.5</td>
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<tr>
<td>Basic Metals</td>
<td>1.8</td>
<td>2.3</td>
<td>3.2</td>
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</tr>
<tr>
<td>Machinery &amp; Equipment</td>
<td>1.9</td>
<td>2.6</td>
<td>3.8</td>
<td>3.4</td>
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<tr>
<td>Motor Vehicles</td>
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<td>2.2</td>
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<tr>
<td>Construction</td>
<td>2.3</td>
<td>2.7</td>
<td>4.5</td>
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<tr>
<td>Wholesale/Retail Trade</td>
<td>3.6</td>
<td>3.9</td>
<td>5.8</td>
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<tr>
<td>Finance &amp; Insurance</td>
<td>2.4</td>
<td>4.4</td>
<td>4.9</td>
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<td>Education</td>
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<td>5.1</td>
<td>6.5</td>
<td>6.6</td>
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<tr>
<td>Avg. Ratio w/ column 1</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Assumes $\gamma_k = \nu_k = 0.1$, $\forall k$. 
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Gains from Trade

▶ To simplify, use $\lambda_k^C = \lambda_k$. We then have

$$GT = \prod_k \lambda_k^{\frac{\psi_k^F}{\varepsilon_k}} \times \prod_k \left( \frac{L_k}{\alpha_k \psi_k^B \bar{L}} \right)^{\theta_k \psi_k^F}$$

where $\psi_k^F \equiv \sum_s \ell_k^F e_s$ and $\psi_k^B \equiv \sum_s \ell_k^B e_s$ are forward and (closed economy) backward Domar weights (Baqaee and Farhi ’21)

▶ Combined with

$$L_k = \alpha_k \sum_s \tilde{\ell}_k^B (\lambda_s e_s + x_s) \bar{L} \geq \alpha_k \sum_s \tilde{\ell}_k^B \lambda_s e_s \bar{L}$$

with $x_s \equiv X_s / \bar{w} \bar{L}$, we then have

$$GT \geq GT^\ast (\lambda) \equiv \prod_k \lambda_k^{\frac{\psi_k^F}{\varepsilon_k}} \times \prod_k \left( \frac{\sum_r \tilde{\ell}_k^B (\lambda) e_r \lambda_r}{\psi_k^B} \right)^{\theta_k \psi_k^F}$$
KLR showed that

\[ \gamma_k \varepsilon_k < 1 \implies P_k \downarrow \text{ as } \lambda_k \downarrow \text{ below one} \implies GT^* > 1 \]

Condition \( \gamma_k \varepsilon_k < 1 \) also guarantees uniqueness

Does UC also guarantee \( GT^* > 1 \) in the current setting?
Convexity of Gains

We show that $G_T^*(\lambda)$ is strictly (log-log) convex in $\lambda$ so if

$$\left. -\frac{\partial \ln G_T^*}{\partial \ln \lambda_i} \right|_{\text{Autky}} \geq 0$$

for all $i$ then $G_T^* > 1$ for any trade pattern.
Gains at Autarky

We have

\[
- \frac{\partial \ln G T^*}{\partial \ln \lambda_i} \bigg|_{\text{Autky}} = \frac{\psi_i^F}{\varepsilon_i} - \sum_k \psi_k^F \theta_k \frac{\partial \ln L^*_k}{\partial \ln \lambda_i} \bigg|_{\text{Autky}}
\]

and

\[
\frac{\partial \ln L^*_k}{\partial \ln \lambda_i} \bigg|_{\text{Autky}} = \frac{\ell_{ki}^B \psi_i^B}{\psi_k^B}.
\]

Using \( \Psi_k \equiv \psi_k^F / \psi_k^B \) for “distortion centrality of sector \( k \)” (Liu ’19) and \( m_i \equiv (1 - \lambda_i) \left[ e_i + \sum_s \alpha_{is} R_s / \bar{L} \right] \) for imports in sector \( i \) as a share of GDP:

\[
\frac{\partial \ln G T^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{\psi_i}{\varepsilon_i} - \sum_k \theta_k \psi_k \ell_{ki}^B
\]
Gains at Autarky: EES in VA

\[
\frac{\partial \ln \text{GT}^*}{\partial m_i} \bigg|_{\text{Autky}} = \psi_i / \varepsilon_i - \sum_k \theta_k \psi_k \ell_{ki}^B
\]

- If EES in VA then \( \mathcal{L}^B = \mathcal{L}^F \) and so \( \psi_k^F = \psi_k^B, \forall k \) plus the UC

\[
\left. \frac{\partial \ln \text{GT}^*}{\partial m_i} \right|_{\text{Autky}} = 1 / \varepsilon_i - \sum_k \theta_k \ell_{ki} > 0, \forall i \implies \text{GT}^* > 1
\]
Gains at Autarky: EES in GO

\[
\frac{\partial \ln G T^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{\psi_i}{\varepsilon_i} - \theta \sum_k \psi_k \ell^B_{ki}
\]

- If EES in GO then \( L^B \neq L^F \) so UC can hold while \( \frac{\partial \ln G T^*}{\partial m_i} \bigg|_{\text{Autky}} < 0 \)

- Assuming \( \varepsilon_i = \varepsilon, \forall i \) and \( \theta_k = \theta, \forall k \) then

\[
\frac{\partial \ln G T^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{1}{\varepsilon} \psi_i - \theta \sum_k \psi_k \ell^B_{ki}
\]

\[\Rightarrow\] imports in sectors with low distortion centrality but high backward distortion centrality can cause welfare losses
Adding Exports

▶ To a first order (around autarky), the gains from trade are

$$\ln GT \approx \sum_k \frac{\Psi_k}{\varepsilon_k} m_k + \sum_{k,s} \theta_k \Psi_k \ell^B_{ks} (x_s - m_s)$$

▶ If EES in VA then

$$\ln GT \approx \sum_k \frac{m_k}{\varepsilon_k} + \sum_s \tilde{\gamma}_s (x_s - m_s),$$

where \( \tilde{\gamma}_s \equiv \sum_k \gamma_k \alpha_k \ell^B_{ks} \)

▶ Higher gains if specialize in sectors with high backward EES
Adding Exports

To a first order (around autarky), the gains from trade are

\[
\ln GT \approx \sum_k \frac{\psi_k}{\varepsilon_k} m_k + \sum_{k,s} \theta_k \psi_k \ell^B_{ks} (x_s - m_s),
\]

With common elasticities then

\[
\ln GT \approx \frac{1}{\varepsilon} \sum_k m_k + \frac{1}{\varepsilon} \sum_k (\psi_k - 1) m_k + \theta \sum_{k,s} \psi_k \ell^B_{ks} (x_s - m_s)
\]

ACR gains \hspace{1cm} DC gains \hspace{1cm} EES gains

DC gains higher if imports mostly in sectors with high DC, which tend to be upstream (Liu '19)

EES gains higher if specialize in sectors with high backward DC
Quantitative Implications

- What are the implications of uniform EES on GT?

- Use world average IO matrix and compute ln GT, ACR, DC and EES gains.

- Regressing ACR gains on ln GT gives share of variance of ln GT explained by ACR gains
Conclusions

- Incorporate EES into quantitative trade models
- Open computational black box: equilibrium and welfare properties
- Sufficient condition for uniqueness

\[ \sum_s \theta_s \ell_{sk}^F \varepsilon_k < 1, \quad \forall k \]

- Nests simpler condition \( \theta_k \varepsilon_k < 1 \) without IO
- IO makes (sufficient) upper bound on \( \theta' s \) much tighter

- UC ensures gains if EES in VA, but not if EES in GO
- Larger GT with specialization in sectors with higher EES upstream